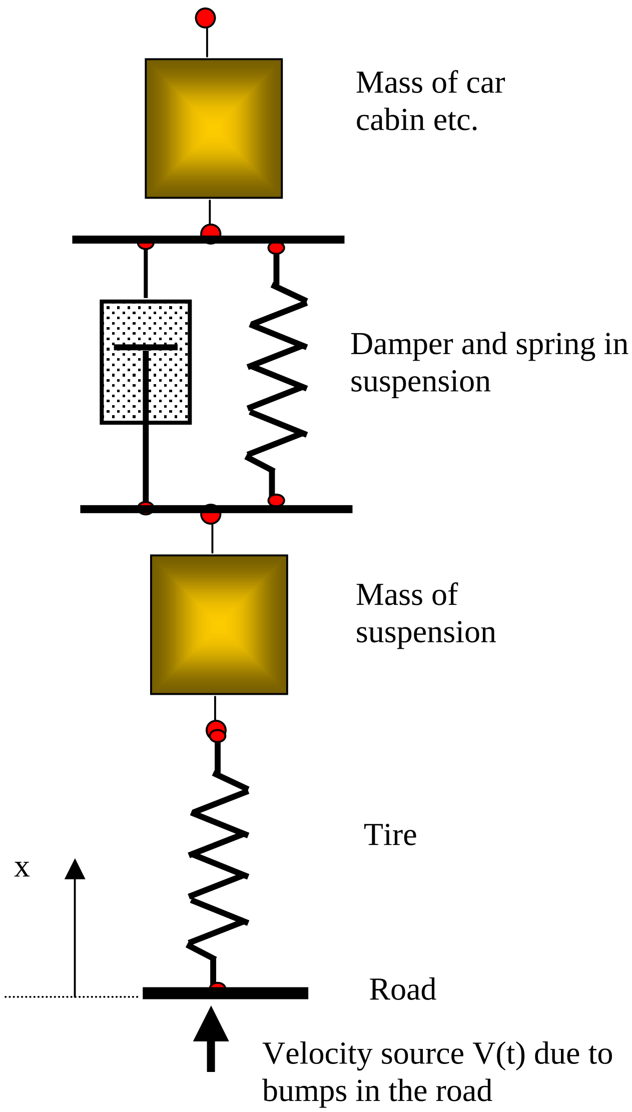
Review Material

You do not have to hand these in. Note that your exam problems may not necessarily be similar to these. These are just to help reinforce some of the concepts you have learned.

1. **Car suspension**



The dynamics of a car suspension mechanism can be modeled as shown in the figure.

The tire is modeled as a spring; the suspension itself is modeled as a mass/spring/damper, atop which sits the rest of the car. The idea is that the bumps in the road (which lead to a *velocity* source at the bottom as shown) will appropriately be damped so that the motion in the car cabin is not too uncomfortable.

The state variables for this system are the stretches of the two springs and the velocities of the two masses.

(1.i) Systematically obtain the first order state equations for this system in the form:

$$\dot{\underline{\mathbf{X}}} = \underline{\mathbf{A}}\,\underline{\mathbf{X}} + \underline{\mathbf{F}}$$

Ignore the velocity source for the rest of the problem, and consider only the *unforced* system equation: $\dot{\underline{\mathbf{X}}} = \underline{\mathbf{A}}\,\underline{\mathbf{X}}$. You can imagine that the car is stationary, for instance.

(1.ii) If the damper in the suspension breaks down by *rigidly* locking, schematically draw a *simplified* model of the broken down system, and obtain the natural frequency of oscillation of this system.

(1.iii) If the damper in the suspension breaks down by completely coming *free*, so that only the spring connects the wheel to the rest of the car, schematically draw a *simplified* model of *this* broken down system. For this case, obtain the simplified first-order state equation $\dot{\underline{\mathbf{X}}} = \underline{\mathbf{A}}\,\underline{\mathbf{X}}$ (again ignoring the velocity source)

(1.iv) Obtain the *second*-order state equations of the form: $\ddot{\underline{\mathbf{Y}}} = \underline{\mathbf{B}}\,\underline{\mathbf{Y}}$ using only the *velocities* of the two masses as the reduced set of state variables for the case in (2.iii)

(1.v) Determine the eigenvalues of the matrix **B** above and thereby obtain the natural frequencies of the system. Assume that the spring constant of both springs is $8 \text{x} 10^6$N/m, the mass of the suspension is 50kg and the mass due to the rest of the car is 800kg. (If this sounds like a light weight car, note that there are three other wheels to take the rest of the load; we are just modeling one of the wheels here).

2. A ball bearing of mass m = 1 kg is gently dropped inside a deep tank containing a viscous fluid. It begins to fall due to gravity. The motion of the bearing is opposed by a drag force exerted by the fluid which in this case is proportional to the velocity and is given by **F**=-C**v**, where C = 0.1Ns/m is a drag coefficient, and **v** is the velocity of the ball bearing. Given that the ball is dropped with initially zero velocity and thereafter falls vertically down, determine the velocity **v**(t) for subsequent times. Plot it. What kind of behavior is this? What is the maximum speed that the ball bearing attains.