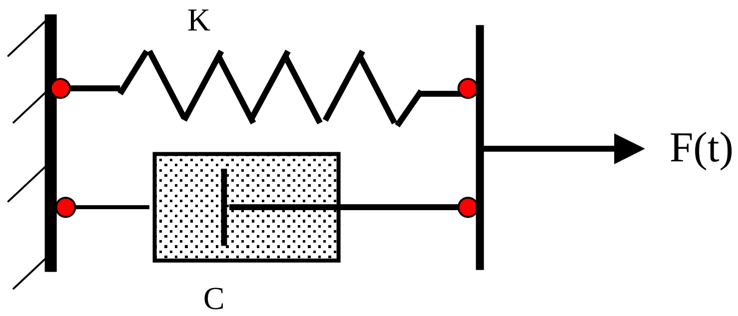
Homework 6

I. A spring-damper system with a force source
The spring-damper system shown above has a force source acting on it.

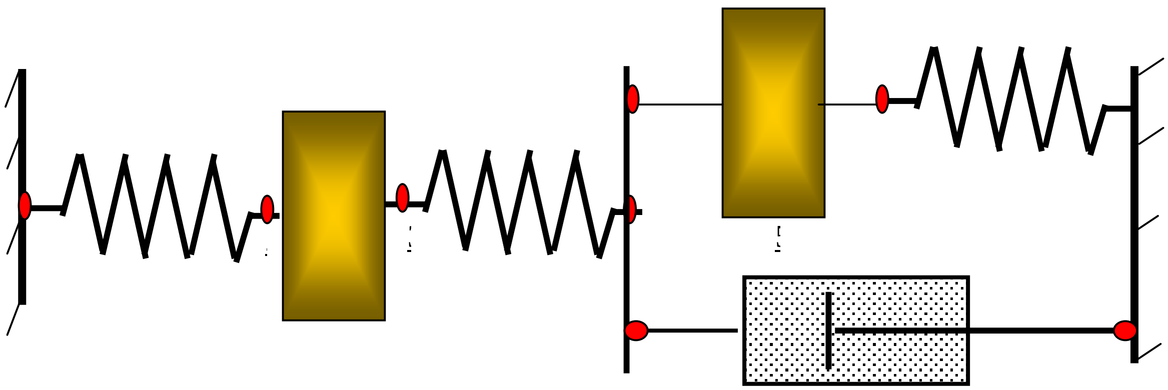


(1.i) [3 points] Systematically obtain the first-order system equation for the state variable: spring stretch.
(1.ii) [2 points] What is the *no-source solution* to the differential equation you obtained above for the spring stretch?
(1.iii) [2 points] If the force source is a *constant* force which acts from time zero, ie:

$$F(t) = F_0, \quad t \geq 0$$

obtain the *source solution* to the differential equation you obtained for this system.
(1.iv) [3 points] Therefore, determine the complete solution for the time-varying behavior of the spring stretch, given that the spring is initially unstretched. Schematically sketch this, marking on the graph the initial and the long-term "steady-state" values of the spring stretch.
(1.v) [2] What is the total energy stored in the spring?
(1.vi) [1] What is total work done by the applied constant force in stretching the spring to its final value?
(1.vii) [2] What is the total energy dissipated in the damper?

II. **A Translational Mechanical System:**
Consider the 3-spring, 2-mass, 1-damper system shown in the figure.



(2.i) [5 points] Systematically obtain the first-order system equation for the state variables: spring stretchs for the three springs and the velocities of the two masses.
(2.ii) [1 point] If the damper becomes ineffective by coming loose, what does the first order system matrix become?
(2.iii) [4 points] Assuming that the damper is ineffective by coming loose, obtain the second order system equation for reduced set of state variables: the velocities of the two masses
(2.iv) [5 points] Given the spring constants for all three springs are 4×10^6 N/m, and the two masses are each 80kg and the damper is ineffective by coming lose, obtain the normal mode solutions and determine the natural frequencies of the system.