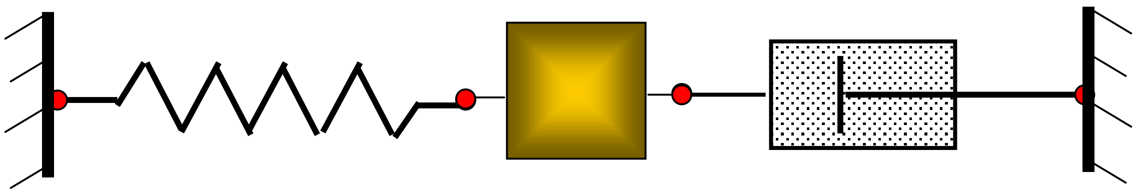
Homework 5

I. Consider the spring-mass-damper system shown.



(1.1) [4 points] Systematically obtain the state equations for the system.

(1.vi) [3 points] Adapt a MATLAB code to solve for the dynamics of this spring-mass-damper system.

Obtain plots for the spring-stretch as a function of time (for 30seconds) for each of the following cases given below.

Mass = 10kg, spring constant = 40N/m for all cases

Case 1: damping coefficient = 8 N.s/m, and:

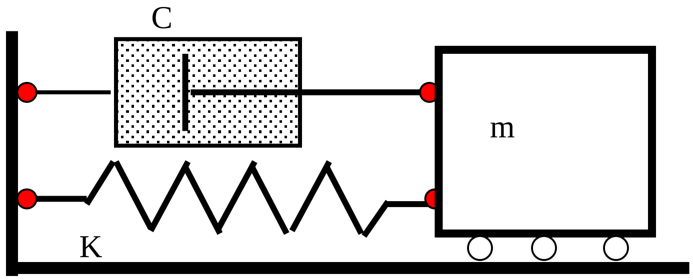
(i) given the initial condition that the spring is stretched by 0.1m, and the initial velocity of the mass is zero
(ii) given the initial condition that the spring is stretched by 0.1m, and the initial velocity of the mass is +1m/s
(iii) given the initial condition that the spring is stretched by 0.1m, and the initial velocity of the mass is -1m/s

Case 2: given the initial condition that the spring is stretched by 0.1m, and the initial velocity of the mass is +1m/s, and:

(i) damping coefficient = 1 N.s/m
(ii) damping coefficient = 8 N.s/m
(iii) damping coefficient = 16 N.s/m

{For each case, there are three sub-cases. Plot the results for all three subcases in the same plot using the MATLAB command ‘hold on’. You should not use the command “close all” since that would close your figure window}

II. [6 points] Consider the spring-mass-damper system shown. Let the mass be pulled to the right by some distance r_o and let go gently. Assume that the floors are frictionless.



For each of these cases determine the kind of motion the system will go into (exponential growth/decay, oscillatory, damped oscillatory?). If the motion is exponential growth/decay, then determine the *time constant* of the growth/decay. If the motion is oscillatory (undamped), then determine the period and natural frequency of oscillations.

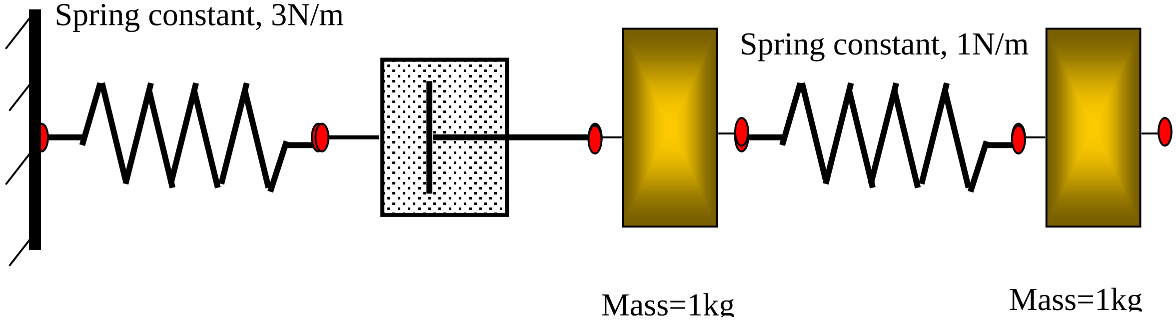
If the motion is damped oscillatory, determine the frequency of the damped oscillations, as well as the time it takes for the oscillation amplitudes to decay away to 1% of their original amplitude.

Case (i): m=10kg, C=0N.s/m (ie no damper), K=100N/m

Case (ii): m=10kg, C=20N.s/m, K=100N/m

Case (iii): m=10kg, C=100Ns/m, K=100N/m

III. Consider the spring-damper-mass-spring-mass system shown.



(3.i) [6 points] Systematically obtain the state equations for the system.

(3.ii) [6 points] Adapt an appropriate MATLAB code to numerically analyze the dynamics of this system for a total of 50 secs from the time the system is set off with some initial conditions given below.

Given: system parameters for springs and masses as in figure.

Case A: damping coefficient = 10 N.s/m, and:

- (i) initial condition that both the springs are stretched by 0.707m, and the initial velocities of both the masses are zero.
- (ii) initial condition that the left spring is stretched by –0.5547m and the right spring by 0.8321m, and the initial velocities of both the masses are zero.
- (iii) initial condition that the left spring is stretched by 0.1m and the right spring by 0.5m, and the initial velocities of both the masses are zero.

Case B: initial condition that both the springs are stretched by 0.707m, and the initial velocities of both the masses are zero.

- (i) damping coefficient 10N.s/m
- (ii) damping coefficient 1e3 N.s/m
- (iii) damping coefficient 1e9 N.s/m

Case C: initial condition that the left spring is stretched by –0.5547m and the right spring by 0.8321m, and the initial velocities of both the masses are zero.

- (i) damping coefficient 10N.s/m
- (ii) damping coefficient 1e3 N.s/m
- (iii) damping coefficient 1e9 N.s/m

Case D: initial condition that the left spring is stretched by 0.1m and the right spring by 0.5m, and the initial velocities of both the masses are zero.

- (i) damping coefficient 10N.s/m
- (ii) damping coefficient 1e3 N.s/m
- (iii) damping coefficient 1e9 N.s/m

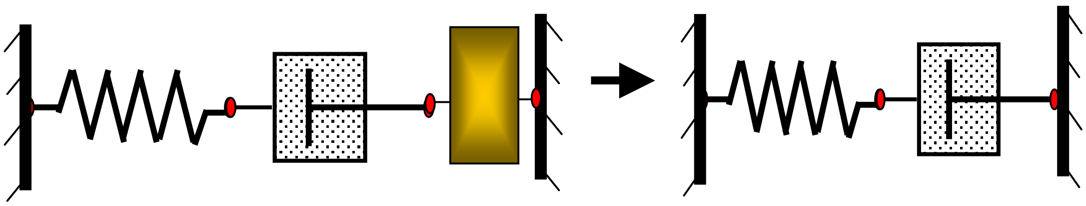
For each of the above cases,
Plot the stretches of the two spring elements as a function of time.
Plot the potential energies in the springs and the kinetic energies in the masses, as well as the *total* mechanical energy of the system, as a function of time.

(3.ii) Answer the following questions based on the numerical results you have obtained:

[1 point] What effect does the damper have on the system as its damping coefficient becomes very large?
[2 points] Looking at the results for the cases with large damping coefficients (B.iii, C.iii, D.iii), what do you observe about the behavior of the system as it is set off with different initial conditions?

IV. Show that the following systems shown on the left can be simplified to the equivalent systems shown on the right:

(4.i) [1 point]



(4.ii) [1 point]