Homework 2
Due:

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[1] **Numerical Integration of Equations of Motion**: The force on a charged particle of mass m and charge q that is in an electric field **E** and magnetic field **B** (you will learn more about electromagnetic fields in later courses) is given by:

$$\mathbf{F} = q\{\mathbf{E} + \mathbf{v} \times \mathbf{B}\}$$

where **v** is the velocity of the particle and **x** denotes vector cross-product.

The MATLAB code *electron.m* has been written to monitor the motion of an electron for one specific electromagnetic field and initial conditions. You should adapt the code given (or re-write a whole new one if you feel like it) and plot the trajectories of an electron:

mass $m = 9.11\text{x}10^{-31}$ kg;
charge $q = -1.6\text{x}10^{-19}$ Coulombs)
that is introduced at **r**(t=0) = 0**i**+0**j**+0**k**
with an initial velocity **v**(t=0) = $2.2\text{x}10^{7}$ **i** ms^{-1}

for the following cases:

(a) a constant electric field **E** = 10**j** kN/Coulomb, and no magnetic field **B**=**0**. (This is already coded. [0 points]
(b) an electric field **E** =-10**j** kN/Coulomb, and a magnetic field **B** = $6.8\text{x}10^{-3}$ **j** kg/Coulomb.sec. [2 points]
(c) no electric field **E** = **0**, and a magnetic field **B** = $6.8\text{x}10^{-3}$ **j** kg/Coulomb.sec. [4 points]
(d) a time varying electric field **E** = 10 sin(ωt) **j** kN/Coulomb where $\omega = 2\text{x}10^{9}\text{s}^{-1}$, and no magnetic field **B**=**0** . [4 points]
(e) an electric field **E** = -10**j** kN/Coulomb, and a time-varying magnetic field **B** = $6.8\text{x}10^{-3}$ sin(ωt) **k** kg/Coulomb.sec where $\omega = 2\text{x}10^{9}\text{s}^{-1}$. [2 points]
(f) no electric field **E** = **0**, and a magnetic field **B** = $6.8\text{x}10^{-3}$ **i** kg/Coulomb.sec. Comment on this trajectory (actually, no numerical computation is needed for this!) [3 points]

Use a run time (total duration) of $8\text{x}10^{-9}$ s, and timesteps of $1\text{x}10^{-11}$ s for all cases. Also, try an improved timestep of $1\text{x}10^{-12}$ s for case (c) and comment on the results.

Provide hardcopies of graphic data output, and the final location of the electron at time $t=8\text{x}10^{-9}$ s for each case.

2. Undamped oscillation: The general solution to the spring-block oscillator we found in class to be:

$$r_x(t) = A\sin\omega t + B\cos\omega t \qquad (*)$$

where $\omega = \sqrt{\frac{K}{m}}$, K is the spring constant and m is the mass of the block; and A and B are constants to be determined through initial conditions. Let the initial conditions be given in general through: $r_x(t=0) = r_{x0}$; $v_x(t=0) = v_{x0}$.

(i) Determine the constants A and B in terms of r_{x0} and v_{x0}. [3 points]

(ii) Using trigonometric identities, show that (*) can also be written as:

$$r_x(t) = E\sin(\omega t + \phi)$$

where $E = \sqrt{A^2 + B^2}$ is called the **amplitude** of the oscillation

and $\phi = \tan^{-1}\frac{B}{A}$ is called the **phase** of the oscillation. [4 points]

3 . Work-energy: You are asked to select a spring that will bring a 150kg package moving at 15m/s to rest in 0.15m from the point of contact. Neglecting friction, what should the spring constant K be to achieve this? [8 points]

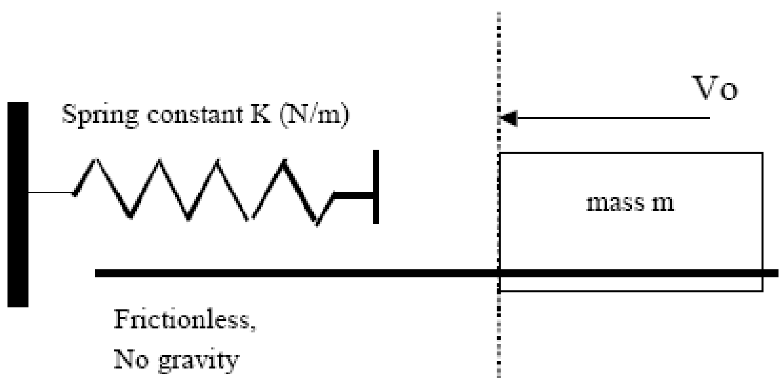

Figure 3