

IV.6 NORMAL MODES

Consider the 2spring-2mass undamped unforced system shown below.

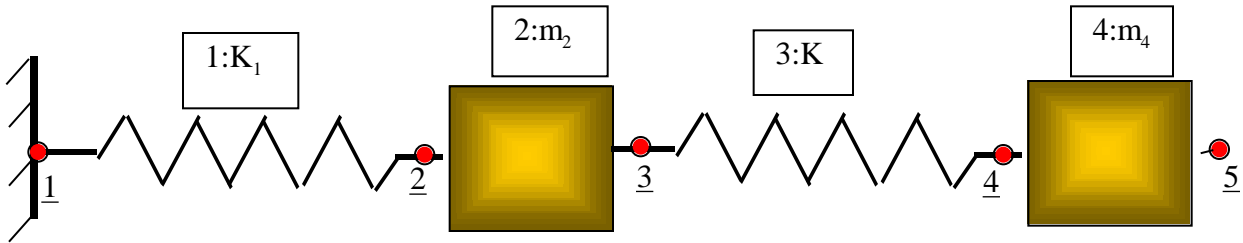


Figure 6.16: A 2spring-2mass system

State Variables:

$$\underline{\mathbf{X}} = \begin{bmatrix} r_{sp1} & v_{m2} & r_{sp3} & v_{m4} \end{bmatrix}^T$$

Geometric Relations:

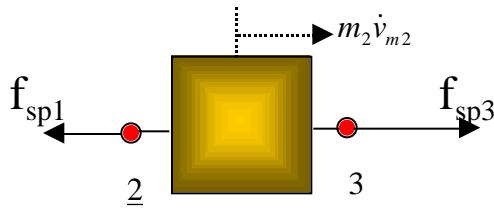
- (i) $x_1 = 0$
- (ii) $r_{sp1} = x_2 - x_1 = x_2$
- (iii) $x_3 - x_2 = \text{constant} \quad \dot{x}_3 = \dot{x}_2 = v_{m2}$
- (iv) $r_{sp3} = x_4 - x_3$
- (v) $x_5 - x_4 = \text{constant} \quad \dot{x}_5 = \dot{x}_4 = v_{m4}$

Equilibrium Relations:

Node 1: just gives me the force exerted on the wall in terms of the spring force in element 1.

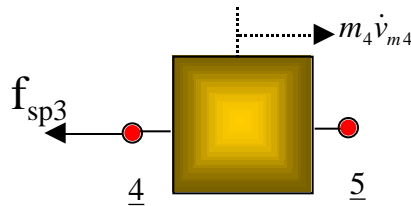
$$\begin{array}{c} \mathbf{f}_{ext1} \\ \leftarrow \bullet \rightarrow \mathbf{f}_{sp1} \\ \underline{1} \end{array} \quad - \quad \mathbf{f}_{ext1} = \mathbf{f}_{sp1}.$$

Nodes 2-3: since there is a mass element between them, we look at the free-body diagram of the mass and its nodes together:



$$(vi) \quad f_{sp3} - f_{sp1} = m_2 \dot{v}_{m2}$$

Nodes 4-5: since there is a mass element between them, we look at the free-body diagram of the mass and its nodes together:



$$(vii) \quad -f_{sp3} = m_4 \dot{v}_{m4}$$

Constitutive Equations:

$$(viii) \quad f_{sp1} = K_1 r_{sp1}$$

$$(ix) \quad f_{sp3} = K_3 r_{sp3}$$

State Equations:

$$\dot{r}_{sp1} \stackrel{(i)}{=} \dot{x}_2 \stackrel{(iii)}{=} v_{m2}$$

$$\dot{v}_{m2} \stackrel{(vi, viii, ix)}{=} \frac{1}{m_2} \{ K_3 r_{sp3} - K_1 r_{sp1} \}$$

$$\dot{r}_{sp3} \stackrel{(iv)}{=} \dot{x}_4 - \dot{x}_3 \stackrel{(v, iii)}{=} v_{m4} - v_{m2}$$

$$\dot{v}_{m4} \stackrel{(vii, ix)}{=} \frac{1}{m_4} \{ -K_3 r_{sp3} \}$$

State Matrix Equation:

$$\begin{array}{l} \dot{r}_{sp1} \\ \dot{v}_{m2} \\ \dot{r}_{sp3} \\ \dot{v}_{m4} \end{array} = \begin{array}{ccccc} 0 & 1 & 0 & 0 & \\ -\frac{K_1}{m_2} & 0 & \frac{K_3}{m_2} & 0 & \\ 0 & -1 & 0 & 1 & \\ 0 & 0 & -\frac{K_3}{m_4} & 0 & \end{array} \begin{array}{l} r_{sp1} \\ v_{m2} \\ r_{sp3} \\ v_{m4} \end{array}$$

Numerical experiment: We can now feed the above to a MATLAB m-file with a Runge-Kutta solver, and get the dynamics of the system for various initial conditions. Here are the m-files *sd_ex2s2m.m* and the associated m-file *rate_fn.m* that do this.

```
%EA3
%
%%-----
%% Example: spring-mass-spring-mass system
%%-----
clear all;
close all;

global A; %share this with function rate_fn which computes Xdot=AX
%%
%% System parameters
```

```

K1    =    3;                %element 1 - spring
m2    =    1;                %element 2 - mass
K3    =    2;                %element 3 - spring
m4    =    1;                %element 4 - mass

%
%State Variables:
%    X(1,:) = stretch of spring element 1
%    X(2,:) = velocity of mass element 2
%    X(3,:) = stretch of spring element 3
%    X(4,:) = velocity of mass element 4

% Initial conditions
i =    1;                    %time index
X(1,1) = 0;
X(2,1) = 1;
X(3,1) = 0;
X(4,1) = 2;

%Energy considerations
PotEnergy(i)=0.5*(K1*X(1,i)*X(1,i)+K3*X(3,i)*X(3,i)); %potential energy in the springs
KinEnergy(i)=0.5*(m2*X(2,i)*X(2,i)+m4*X(4,i)*X(4,i)); %kinetic energy in the masses
TotEnergy(i)=PotEnergy(i)+KinEnergy(i);                %total energy in the system

%time parameters
start_time = 0;
dt        =    0.01;        %time step chosen
final_time =    20;         %time to end calculation
time(i)   =    start_time;  %let us keep a vector of times for plotting

%State matrix
A = [0,          1,      0,      0;
     -K1/m2,     0,      K3/m2, 0;
     0,          0,      -1,     0,          1;
     0,          0,      0,      -K3/m4,0]; % for use in equation Xdot = A X

% Integrate equations of motion using Runge-Kutta algorithm

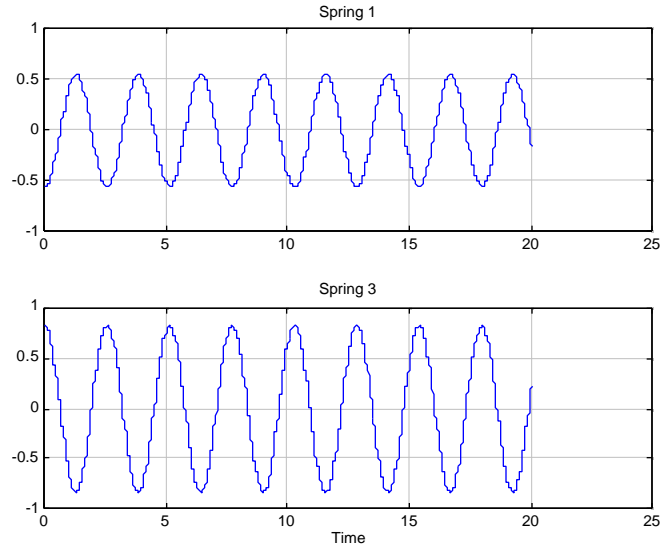
for t = start_time+dt:dt:final_time

    k1 = rate_fn(X(:,i)); % current rate
    k2 = rate_fn(X(:,i) + dt*k1/2); % estimated mid-point rate
    k3 = rate_fn(X(:,i) + dt*k2/2); % even better mid-point rate
    k4 = rate_fn(X(:,i) + dt*k3); % excellent end-point rate
    X(:,i+1) = X(:,i) + dt*(k1/6 + k2/3 + k3/3 + k4/6); % use weighted average of rates
    time(i+1) = time(i) + dt;
    i = i+1;

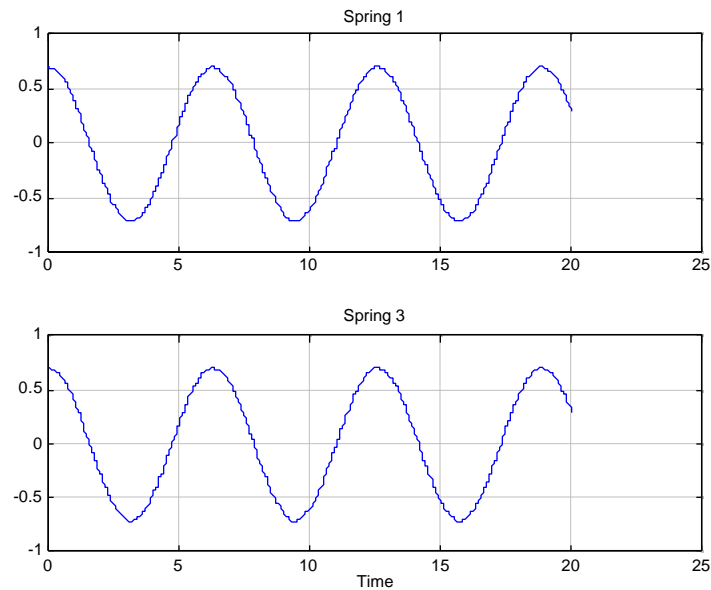
    %energy
    PotEnergy(i)=0.5*(K1*X(1,i)*X(1,i)+K3*X(3,i)*X(3,i));
    KinEnergy(i)=0.5*(m2*X(2,i)*X(2,i)+m4*X(4,i)*X(4,i));
    TotEnergy(i)=PotEnergy(i)+KinEnergy(i);
end

```


(b) For initial conditions: $X(0) = [-0.5547 \quad 0 \quad 0.8321 \quad 0]'$ we find:



(c) And for initial conditions: $X(0) = [0.7071 \quad 0 \quad 0.7071 \quad 0]'$ we find:



Remarks:

- (i) From case (a) the behavior of the two springs seems rather complicated.
- (ii) But from (b,c) it appears that if I start the system off just right, I get a nice oscillatory behavior for the spring stretches (and also for the velocities of the

masses as you can check by plotting them as well). Nice in the sense that these can be represented by *sines* and *cosines* (as you can see from the plots).

Indeed we can *measure* from the graphs that it should be possible to get an analytical solution to this 2spring-2mass system, and that this solution would for case (b) look like:

$$\begin{aligned} r_{sp1}(t) &= -0.5547 \cos \omega_1 t \\ r_{sp3}(t) &= 0.8321 \cos \omega_1 t \end{aligned} \quad (\alpha)$$

where the angular frequency of oscillation takes the value: $\omega_1 = \sqrt{6}$ as we shall see shortly, or as you can measure from the graphs (how would you do this?).

Similarly for case (c) the analytical solution should look something like:

$$\begin{aligned} r_{sp1}(t) &= 0.7071 \cos \omega_2 t \\ r_{sp3}(t) &= 0.7071 \cos \omega_2 t \end{aligned}$$

where the angular frequency of oscillation now takes the value: $\omega_2 = 1$.

In each case, the two springs oscillate with the same frequency. Such solutions where all the dynamic variables of a system oscillate *harmonically* (like a sine or a cosine) with the same frequency, are called the **normal mode** solutions of the system. We will shortly see how to actually get these normal mode solutions analytically for any undamped system. In fact, I was able to give you the correct set of initial conditions to set these systems off into their normal modes only because I knew how to get these solutions analytically! Otherwise you will have to do a lot of numerical experiments by trying various values for the initial conditions till you accidentally stumble upon the solution.

- (iii) Note that in case (b) when spring 1 stretches, spring 3 compresses. We say that the springs are *out of phase* with respect to each other. Note that in (α) :

$\cos(\omega_1 t + \pi) = -\cos(\omega_1 t)$ and so it is the phase (refer to Chap.3 or Pset 2 if you have forgotten what we defined as phase) of the two spring oscillations that differ by 180° . For case (c), both springs are *in phase* and they oscillate together.

- (iv) The frequencies of oscillation of the normal modes are called the *natural frequencies* of the system (natural because there is no force sources here). Normal modes are important for several reasons. We need to worry about the system going into resonance if there are external forces acting on the system that might lead to catastrophically large motions of the system. Also, the funny looking solution for case (a) is not really beyond analytical reach. It turns out that it is just a *linear*

combination of the normal mode solutions! We will now have to look into this curious behavior analytically.

Analytical Determination of Normal Modes:

Now that we know that it is possible to set off our undamped system into its *normal modes* where the whole system oscillates harmonically, we need to figure out how to determine these normal modes for complex systems of springs and masses. To do this, as always when we try to get analytical solutions, we swap out some of our state variables. As before, I will swap out the velocities of the masses in favor of the spring stretches. This means we have to do some more algebra, and as before, we will end up with *second order* differential equations. However, now we do not have just one spring stretch to worry about, but two (and for a more complex system, I might have several). Therefore, it would be a good idea to adopt a procedure that is already familiar to you from *EAI:Linear Algebra*. To help clarify matters, I will walk you through this algebra both in expanded form (left column) and in matrix form (right column).

Let us recall the *first-order* state equation we found for this system: $\dot{\underline{X}} = \underline{A}\underline{X}$ which in expanded form looks like:

$$\begin{array}{r} \dot{r}_{sp1} \\ \dot{v}_{m2} \\ \dot{r}_{sp3} \\ \dot{v}_{m4} \end{array} = \begin{array}{cccc} 0 & 1 & 0 & 0 \\ -\frac{K_1}{m_2} & 0 & \frac{K_3}{m_2} & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -\frac{K_3}{m_4} & 0 \end{array} \begin{array}{l} r_{sp1} \\ v_{m2} \\ r_{sp3} \\ v_{m4} \end{array} \quad (\zeta)$$

First, I will create a *reduced* set of state variables containing only the spring stretches:

$$\underline{Y} = \begin{array}{l} r_{sp1} \\ r_{sp2} \end{array}$$

Then, our *second-order* form for the state equations will be an expression of the form:

$$\ddot{\underline{Y}} = \underline{B}\underline{Y}$$

where the superposed double-dot is shorthand for *two* time derivatives; and \underline{B} is called the *second-order state matrix* for the system.

To determine the \underline{B} matrix, we need to start with our first order state equations, and play with them till we eliminate the state variables we don't want. Here, we need to get rid of

the velocities of the masses. A little algebra is called for (the pointers refer to the row number of the equations in the first-order matrix state equation (ç) above):

$$\begin{aligned} \ddot{r}_{sp1} &= \overset{(1)}{\dot{v}_{m2}} = -\frac{K_1}{m_2} r_{sp1} + \frac{K_3}{m_2} r_{sp3} \\ \ddot{r}_{sp3} &= -\overset{(3)}{v_{m2}} + \overset{(2,4)}{v_{m4}} = -\frac{K_1}{m_2} r_{sp1} + \frac{K_3}{m_2} r_{sp3} + \frac{K_3}{m_4} r_{sp3} \end{aligned}$$

Therefore:

$$\begin{aligned} \begin{matrix} \ddot{r}_{sp1} \\ \ddot{r}_{sp3} \end{matrix} &= \begin{matrix} -\frac{K_1}{m_2} & \frac{K_3}{m_2} \\ \frac{K_1}{m_2} & -\frac{K_1}{m_2} - \frac{K_3}{m_4} \end{matrix} \begin{matrix} r_{sp1} \\ r_{sp3} \end{matrix} \quad \rightarrow \quad \ddot{\underline{Y}} = \underline{B}\underline{Y} \quad () \\ \text{Expanded form} & \qquad \qquad \qquad \text{matrix form} \end{aligned}$$

which is our desired second-order state equation. To obtain the normal-mode solutions, we see if the second-order state equation admits harmonic (sine and cosine) solutions (which as we have seen before are related to complex exponentials). Therefore, we *seek* solutions to the second-order state equations of the form:

$$\begin{aligned} \begin{matrix} r_{sp1} \\ r_{sp3} \end{matrix} &= \begin{matrix} 1 \\ 2 \end{matrix} e^{j\omega t} \quad \rightarrow \quad \underline{Y} = \underline{c} e^{j\omega t} \quad (\dagger) \\ \text{Expanded form} & \qquad \qquad \qquad \text{matrix form} \end{aligned}$$

where the c 's are constants (related to the initial conditions with which I must set the system off to get the appropriate normal mode behavior).

The second-order state equation () then becomes:

$$\begin{aligned} -\omega^2 \begin{matrix} 1 \\ 3 \end{matrix} &= \begin{matrix} -\frac{K_1}{m_2} & \frac{K_3}{m_2} \\ \frac{K_1}{m_2} & -\frac{K_1}{m_2} - \frac{K_3}{m_4} \end{matrix} \begin{matrix} 1 \\ 3 \end{matrix} \quad \rightarrow \quad -\omega^2 \underline{c} = \underline{B}\underline{c} \\ \text{Expanded form} & \qquad \qquad \qquad \text{matrix form} \end{aligned}$$

This can be recast in the form:

$$\begin{array}{ccc} -\frac{K_1}{m_2} - \lambda & \frac{K_3}{m_2} & \\ \frac{K_1}{m_2} & -\frac{K_3}{m_2} - \frac{K_3}{m_4} - \lambda & \end{array} \begin{array}{c} 1 \\ 3 \end{array} = \begin{array}{c} 0 \\ 0 \end{array} \quad \Rightarrow \quad [\underline{\mathbf{B}} - \lambda \underline{\mathbf{I}}] \underline{\mathbf{x}} = \underline{\mathbf{0}} \quad (7)$$

Expanded form

matrix form

where $\lambda = -\omega^2$ is a simple change of variables that helps put the matrix equation in a form that might be familiar to you from EA1?

Now, for what $\underline{\mathbf{x}}$ will the above linear *homogeneous* equations be satisfied? Clearly, one *trivial* solution is that $\underline{\mathbf{x}} = \underline{\mathbf{0}}$. That is, nothing happens and the system does not move at all. That is why we call this a trivial solution. But is there any non-trivial solution to this system of equations? From EA1, you must know that a non-trivial solution to the above system of linear algebraic equations is possible if the *determinant* of the matrix multiplying $\underline{\mathbf{x}}$ is zero. That is:

$$\begin{vmatrix} -\frac{K_1}{m_2} - \lambda & \frac{K_3}{m_2} \\ \frac{K_1}{m_2} & -\frac{K_3}{m_2} - \frac{K_3}{m_4} - \lambda \end{vmatrix} = 0 \quad \Rightarrow \quad \det[\underline{\mathbf{B}} - \lambda \underline{\mathbf{I}}] = 0$$

Expanded form

matrix form

where $\underline{\mathbf{I}}$ is the identity matrix. This provides us a polynomial equation that (equivalently) must satisfy. For a given system of springs and masses, there will be a discrete set of values ω_i (equivalently λ_i) for which this equation will be satisfied. Corresponding to each ω_i (equivalently λ_i), there will an associated set $\underline{\mathbf{x}}_i$ which gives us the initial conditions with which we must set off the system so that it goes into the corresponding normal mode. It is worth reiterating that once we figure out the second-order state matrix $\underline{\mathbf{B}}$ for a system, all we need to do is find that set of ω_i (equivalently λ_i) and $\underline{\mathbf{x}}_i$ that satisfy: $\lambda_i \underline{\mathbf{x}}_i = \underline{\mathbf{B}} \underline{\mathbf{x}}_i$. The ω_i (equivalently λ_i) give us the frequency of the normal modes and the $\underline{\mathbf{x}}_i$ give us the initial conditions with which we must set off the system so that it goes into the corresponding normal mode.

*Terminology: The values λ which satisfy this equation: $\lambda \underline{v} = \underline{B} \underline{v}$ are called the **eigenvalues** of the matrix \underline{B} , and the associated \underline{v} are called the **eigenvectors**.*

For the case that we were considering in our numerical experiments: $m_2=m_4=1$, and $K_1=3, K_3=2$, the second-order state equation becomes:

$$\begin{bmatrix} \ddot{r}_{sp1} \\ \ddot{r}_{sp3} \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} r_{sp1} \\ r_{sp3} \end{bmatrix}.$$

We require: $\det[\underline{B} - \lambda \underline{I}] = 0$ which becomes.

$$\begin{vmatrix} -3 - \lambda & 2 \\ 3 & -4 - \lambda \end{vmatrix} = 0 \rightarrow \lambda^2 + 7\lambda + 6 = 0.$$

The two roots of which are $\lambda_1 = -6$ and $\lambda_2 = -1$. Therefore the corresponding natural frequencies are given by $\omega^2 = -\lambda$ $\omega_1 = \sqrt{6}$, $\omega_2 = 1$, which is what we guessed (measured) earlier from the numerical experiments with MATLAB.

Let us see what the corresponding values of \underline{v} are.

For $\lambda = -6$, from $(-)$ we find:

$$\begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 3v_1 + 2v_3 = 0 \\ 3v_1 + 2v_3 = 0 \end{bmatrix} \rightarrow v_3 = -\frac{3}{2} v_1$$

Since one of our equations is linearly dependent on (in this case, exactly the same as) the other, you know from EA1 that there is an infinity of solutions, but they must all obey the relation on the right. Therefore, the corresponding eigenvector is:

$$\underline{v}_{(\lambda=-6)} = \begin{bmatrix} v_1 \\ v_3 \end{bmatrix} = \begin{bmatrix} -0.5547 \\ 0.8321 \end{bmatrix}$$

The above gives me one possible way for me to set off the system to get it to go into this normal mode with angular frequency $\omega_1 = \sqrt{6}$. (Other possible solutions are scalar multiples of this one.) Now, why did I choose these funny looking numbers rather than some round numbers such as: $v_1 = 1$; $v_3 = -3/2$ which also work? For no particular reason, except that it is customary to express the eigenvectors as *unit* vectors, ie to choose them here such that: $\sqrt{v_1^2 + v_3^2} = 1$.

Therefore the first normal mode solution that we have obtained is:

$$\underline{Y}_1 = \begin{bmatrix} -0.5547 \\ 0.8321 \end{bmatrix} e^{j\omega_1 t}; \quad \omega_1 = \sqrt{6}.$$

We can repeat the above for the other eigenvalue $\lambda_2 = -1$, and we find that:

$$\begin{bmatrix} -2 & 2 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} -2x_1 + 2x_3 = 0 \\ 3x_1 - 3x_3 = 0 \end{bmatrix} \rightarrow x_3 = x_1$$

Therefore, the corresponding eigenvector is:

$$\underline{Y}_{(\lambda=-1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.7071 \\ 0.7071 \end{bmatrix}$$

where once again I have chosen a unit eigenvector (from among an infinite set which are all scalar multiples of one another).

Therefore the second normal mode solution for this system is:

$$\underline{Y}_2 = \begin{bmatrix} 0.7071 \\ 0.7071 \end{bmatrix} e^{j\omega_2 t}; \quad \omega_2 = 1.$$

We have now figured out two normal mode solutions to () which are in complex exponential form. It is a trivial matter to convince yourself that:

$$\underline{Y}_3 = \begin{bmatrix} -0.5547 \\ 0.8321 \end{bmatrix} e^{-j\omega_1 t}; \quad \omega_1 = \sqrt{6}; \quad \text{and} \quad \underline{Y}_4 = \begin{bmatrix} 0.7071 \\ 0.7071 \end{bmatrix} e^{-j\omega_2 t}; \quad \omega_2 = 1,$$

are also solutions to (). Just refer back to equation (†) and you will see that if the positive exponential works, so does the negative exponential! And from the fact that we are dealing with linear differential equations (see the web notes on Euler's DE), we note that any *linear combination* of the above solutions is also a solution to the DE (). That is, the *complete* solution to the second-order system state equations () is:

$$\underline{Y} = \begin{bmatrix} -0.5547 \\ 0.8321 \end{bmatrix} \{a_1 e^{j\omega_1 t} + b_1 e^{-j\omega_1 t}\} + \begin{bmatrix} 0.7071 \\ 0.7071 \end{bmatrix} \{a_2 e^{j\omega_2 t} + b_2 e^{-j\omega_2 t}\}$$

where the a's and b's are obtained from initial conditions. How do you know that this is the *complete* solution to ()? Right now, you do not. But take it from me that it is, and you will learn more about these things in EA4.

One more thing. We can now recast the above in the form of sines and cosines using the relationship between complex exponentials and trigonometric functions to get:

$$\underline{Y} = \begin{matrix} r_{sp1} \\ r_{sp3} \end{matrix} = \begin{matrix} -0.5547 \\ 0.8321 \end{matrix} \{A_1 \cos \omega_1 t + B_1 \sin \omega_1 t\} + \begin{matrix} 0.7071 \\ 0.7071 \end{matrix} \{A_2 \cos \omega_2 t + B_2 \sin \omega_2 t\} \quad (\text{¥})$$

where the A's and B's (related to the a's and b's) are constants to be determined from initial conditions. (Recall we did this for the single spring-block oscillator).

Remarks:

- (a) The complete solution to () is (¥), and in fact the funny looking solution we saw in our numerical experiments (case a) is actually contained in (¥). The funny looking response is just a sum of a bunch of sines and cosines!

[For the curious among you: Try this. Take (¥) as the general solution to (), and use the initial conditions $\underline{X}(0) = [0.1 \ 0 \ -0.5 \ 0]^T$ for case (a) in the numerical experiment. Determine the corresponding constants A's and B's. Plot the analytical solution along with the numerical to see if this works.]

- (b) You might wonder whether I am not cheating here. Just a little bit! I did tell you earlier that the eigenvectors had to do with initial conditions (or how I start the system off to get it into its normal modes). And now, I have introduced a whole bunch of new constants to be obtained from initial conditions, and the eigenvectors are still apparently hanging around in (¥)! In truth, the eigenvectors are not really initial conditions. Rather, they represent the *relative amplitudes of normal mode oscillation* of the various state variables of our system. Remember that we were not able to determine the eigenvectors uniquely, and that any scalar multiple of the eigenvector was also an eigenvector. For instance, here the eigenvectors actually only give me the *ratios* of the stretches of spring elements one and three, not their actual values. You will learn in EA4 and later that eigenvectors have a lot more meaning than we can appreciate at this point in our journey.

But to see that the eigenvectors are actually related to the relative amplitudes of normal mode oscillations, I suggest that you return to the numerical experiment and run the code with the following initial conditions: $\underline{X}(0) = [0, 1, 0, -0.5]^T$ and $\underline{X}(0) = [0, 0.5, 0, 1]^T$. That is, the initial conditions now are on the velocities of the two masses and

the springs are initially unstretched. What do you observe for the stretches of the springs in terms of both relative amplitudes and frequency?

- (c) Note that seeking complex exponential solutions is a lot easier than seeking sines and cosines (in terms of the algebraic manipulations we have to make). However, at the end of it all, we return to the trigonometric form of the results. This is a favorite trick in engineering analysis. Rather than seek solutions involving sines first, and then cosines next, and putting them all together (as linear combinations), we seek *complex* exponential solutions, and then recognize that at the end we can simply extract the sine and cosine forms!
- (d) MATLAB has a built-in function called 'eig' which does the necessary calculations to determine the eigenvalues and eigenvectors of a matrix. The command:

$$[\text{Evector}, \text{Evalue}] = \text{eig}(\text{B})$$

produces a diagonal matrix Evalue of eigenvalues and a full matrix Evector whose columns are the corresponding eigenvectors of the matrix B.

Here is an m-file that calculates the eigenvalues and eigenvectors for our system.

```
%EA3-Fall 1999
%% DETERMINATION OF NORMAL MODES
%%-----
clear all;
close all;

%Define Y (reduced set of state variables for second-order form)
% Y(1) = rsp1
% Y(2) = rsp2

%Enter parameters of system
K1 = 3;
m2 = 1;
K3 = 2;
m4 = 1;

%Enter B matrix (Ydotdot = BY -- second order form)
B = [-K1/m2      K3/m2;
      K1/m2      -K3/m2-K3/m4]

%
[Evector,Evalue] = eig(B);

disp('The Eigenvalues are:'), disp(diag(Evalue))
disp('The Eigenvectors are:'), disp(Evector)
```

- (e) This is more math than you can possibly digest in one day. Go over it very carefully to make as much sense of it as you can at this point. You will revisit this topic in EA4 in depth. The topic of eigenvalues and eigenvectors is very, very important. The analyses of physical phenomena in several diverse fields turn out to

end up as eigenvalue problems. Right now, all I want you to get from here is an appreciation of the eigenvalue analysis procedure. You will work through one problem for your homework and that will help fix some of these ideas. Also, always keep in mind the physical meaning of the normal mode solutions.

- (f) By the way, there is one important question that I have side stepped. How many normal mode solutions are there for a given system? This is important because it tells me how many natural frequencies I have to worry about. I will continue to shy away from answering this question for now!