COMPLEX EXPONENTIALS

Consider the function: f(t)
Let it be a well-behaved function (for which derivatives of all orders necessary exist everywhere).

Functions like f(t)= sinωt; f(t) = $e^{\omega t}$ are acceptable.

Functions like |ωt| are not.

Note that in Calculus you learnt (or should have learned) that such functions can be expanded in a Taylor series as follows:

$$f(t) = f(0) + \frac{df}{dt}(0)\frac{t}{1!} + \frac{d^2 f}{dt^2}(0)\frac{t^2}{2!} + \frac{d^3 f}{dt^3}(0)\frac{t^3}{3!} + \ldots + \frac{d^n f}{dt^n}(0)\frac{t^n}{n!} + \ldots$$

where n!=n.(n-1).(n-2)…(3).(2).(1) is read as (n-factorial);
and the derivatives are all evaluated at t=0. That is, most well-behaved functions can be expressed in the form of a power series!

For example, it should be straightforward for you to obtain the following Taylor series expansions for some common functions:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} \ldots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$

(I am using x instead of t now and soon will use something else, but these are just symbols representing the *argument* of the function under consideration).

Now the meaning of a complex exponential function becomes clear. I just need to set x=jθ in the above to get:

$$e^{j\theta} = 1 + (j\theta) + \frac{(j\theta)^2}{2!} + \frac{(j\theta)^3}{3!} + \frac{(j\theta)^4}{4!} + \frac{(j\theta)^5}{5!} + \frac{(j\theta)^6}{6!} + \frac{(j\theta)^7}{7!} \ldots$$

$$= \left\{1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \ldots\right\} + j\left\{\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \ldots\right\}$$

$$= \cos\theta + j\sin\theta$$

where I have used the fact that $j = \sqrt{-1};\ j^2 = -1;\ j^3 = j^2 j = -j;\ j^4 = +1;\ldots$
That is, the complex exponential is related to the trigonometric funtions: *sine* and *cosine.*
We have just derived **Euler's formula**.