Review Problem Set 2

- You do not have to hand this in, Solutions to these problems will not be provided.
- Final exam is not necessarily restricted to only these kinds of problems. These problems do not cover the first half of the course, and are provided only to get you more comfortable with the Rayleigh-Ritz method.

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1. Column Buckling using Rayleigh-Ritz: Determine the critical buckling load of a column which is hinged at both its ends and has a cross-section whose moment of inertia varies as:

$$I_z = \begin{cases} I_1\left(1 + \dfrac{3x}{L}\right) & \text{for } 0 \le x \le \dfrac{L}{2} \\ I_1\left(4 - \dfrac{3x}{L}\right) & \text{for } \dfrac{L}{2} \le x \le L \end{cases}$$

where $I_1 = 4.9\text{x}10^{-6}$ m^4 is the moment of inertia of the end cross-sections. The Young's modulus of the beam is E = 71 GPa, the length L = 3m.

2. Column Buckling: Can a tree buckle under its own weight? Show how you would approach this problem to determine the critical buckling load. Make any reasonable assumptions.

3. Stretching of a Rod Using Multi-part Rayleigh-Ritz: A stepped bar, which is subject to a tensile load P, has three segments of equal length and area 4 A_0, 2 A_0 and A_0 as shown in the figure:

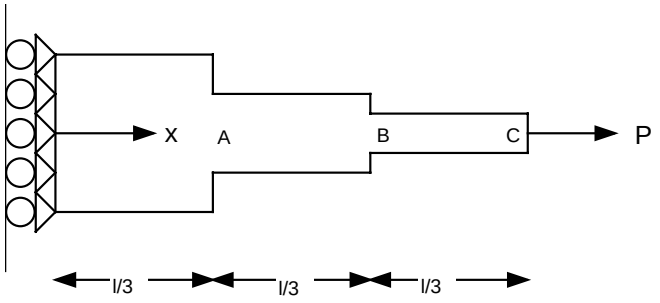

Figure: Stepped Rod

Assume that the displacements in each of the three segments varies linearly with x. (i) What are the displacements in each segment in terms of the nodal displacements at points A, B and C? (ii) Using the Rayleigh-Ritz procedure, get the stiffness matrix and the load vector for the problem. (iii) Solve for the nodal displacements given: P = 5kN, A_0 = $0.0022m^2$, L = 1m, and E = 190 GPa .

4. Plate bending: A thin, rectangular plate of uniform thickness h=0.005m, and sides a=0.3m and b=0.5m is clamped on the sides x=0 and x=a and is simply supported on the sides z=0 and z=b. The plate, which is made of a linear elastic isotropic material with Young's modulus E=71GPa and Poisson's ratio ν=0.211, carries a concentrated load P=50,000N at its midpoint. (i) What is a suitable one term polynomial Rayleigh-Ritz function for this problem? (ii) What is the displacement of the mid-point of the plate?