Problem Set 6

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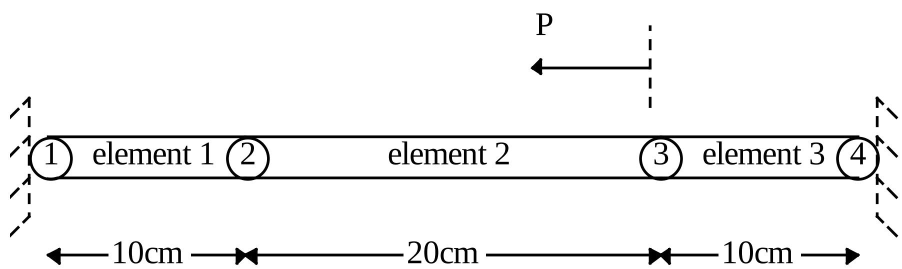
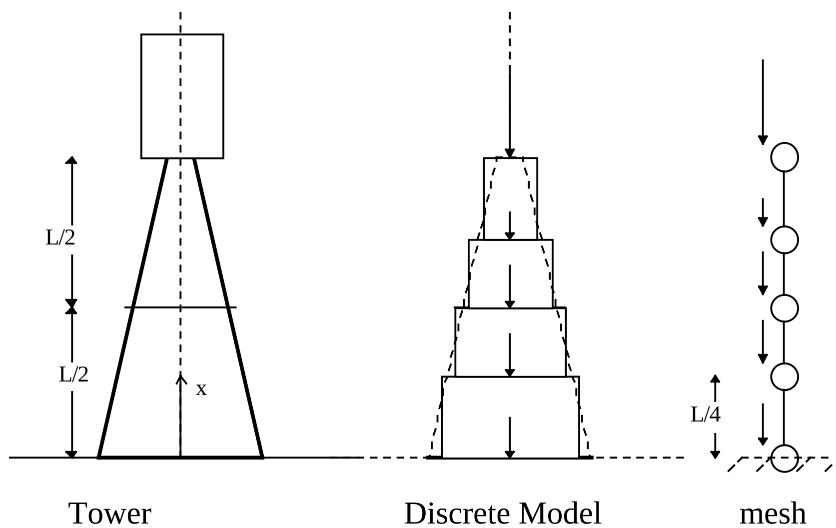
1. Stretching of a Stepped Rod: Consider the three-stepped rod system shown in the figure. All three rod segments are made from the same material whose Young's modulus is $E = 2.1\times10^{11} Nm^{-2}$. Elements 1 and 3 have an area of $0.2m^2$ and element 2 has an area of $0.3\ m^2$. An axial force P of magnitude 60,000N acts at node 3 along the negative x-direction. Both nodes 1 and 4 are constrained not to displace.



[8 points] Analyze this structure using the FEM approach by hand (ie not using ANSYS). Obtain the stresses in all the elements and all the nodal displacements.

2. A Tapered Tower: The observation tower shown in figure consists of a tapered base of length L=100m whose area varies as $A(x) = A_o \{1\text{-}0.5x/L\}$ where $A_o = 1m^2$. Atop the base sits the observation room. The first half of the tower ($0 < x < 0.5L$) is made of steel of Young's modulus $E_{st} = 2.1\times10^{11}\ Nm^{-2}$ and density $\rho_{st} = 8100kg/m^3$ and the rest of the tower is made of aluminum of Young's modulus $E_{al} = 0.7\times10^{11}\ Nm^{-2}$ and density $\rho_{al} = 2700kg/m^3$. The observation room and its contents weigh 100,000N.

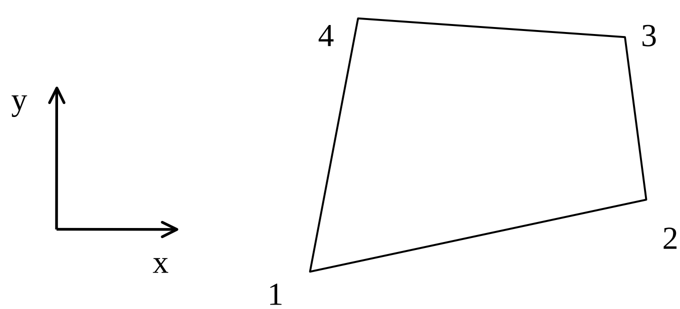


[12 points] Model the tower (without the observation room) as being made of 4 elements of equal length. Let the areas of these elements be the average area of the corresponding tapered section. Model the weights of each segment as being lumped at the corresponding nodes below. The observation room is removed and its weight is assumed to act as shown on the top element. Using this "finite element" model of the tower, obtain the displacements and the stresses in the tower. Again, use the FEM approach by hand (do not use ANSYS).

Remark: You can get even better results by *refining* the mesh, that is by using say 8 elements rather than 4. This way, you will be modeling the taper of the tower a little better. Try this if you are adventurous, but you do not have to hand this in.

NOTE: Read the web notes of finite elements in 2D and 3D before answering the next question

3. The Bi-Linear Quadrilateral Element: There are situations when bilinear quadrilateral two-d elements with 4-nodes are useful.



The elemental nodal displacement vector is $\mathbf{d}_e^{\mathrm{T}} = \{u_1 \quad v_1 \quad u_2 \quad v_2 \quad u_3 \quad v_3 \quad u_4 \quad v_4\}$

where there are 2 degrees of freedom at each node . Assuming that the displacements inside the element are given by the same bilinear function for both the x- and y-displacements:

$$u_x(x,y) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 xy$$
$$u_y(x,y) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 xy$$

The displacement function vector can therefore be expressed as: $\begin{Bmatrix} u_x(x,y) \\ u_y(x,y) \end{Bmatrix} = \mathbf{N_e d_e}$, where

$\mathbf{N_e}$ is the shape matrix.

And the strain matrix can therefore be written as: $\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_x}{\partial x} \\ \dfrac{\partial u_y}{\partial y} \\ \dfrac{\partial u_x}{\partial y} + \dfrac{\partial u_y}{\partial x} \end{Bmatrix} = \mathbf{B_e d_e}$

[5 points] Determine the shape function matrix $\mathbf{N_e}$ as well as the $\mathbf{B_e}$ matrix for the strains for the 4-node quadrilateral element. Is the B-matrix constant for the 4-node quad element? When might we want to use this kind of an element?

Note: Those of you who are familiar with MAPLE may want to do the above algebra using symbolic manipulation. This is an opportunity to learn how to do algebra (not just calculations) on a computer. Look at the code **trielement.m** that I have written to derive the fem matrices for a linear triangular element, and modify it for the linear quadrilateral element. If you are not familiar with symbolic manipulation, just outline the procedure, ie do not do a huge amount of algebra. I just want you to see what goes into the B-matrix. Just provide the structure of the B-matrix (how many slots, what goes into each slot…)