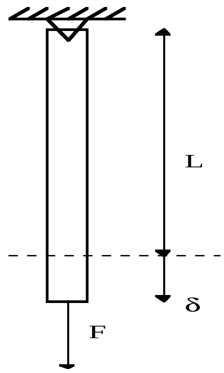
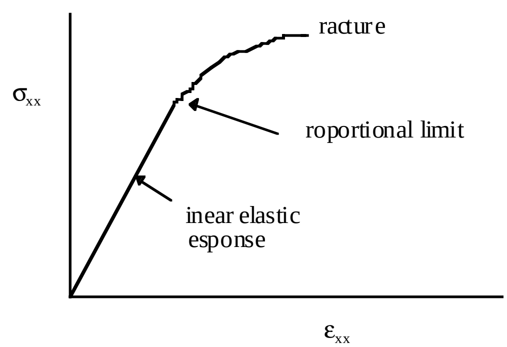
3.1 MATERIAL RESPONSE



Consider a cylinder made of steel. Let us apply a force F to it. It deforms by a certain amount. However, if we make a geometrically identical specimen out of rubber and apply the same force F, it deforms by a much greater amount. Clearly, the deformation of a body subject to a certain applied force is not only dependent on the magnitude of the applied force, but also depends on the type of material the body is made of. Suppose that our steel cylinder is uniform and has an initial length L and cross-sectional area A. Let us do the following experiment. Let us gradually increase the applied load F and monitor the deformation ΔL.

The stress in the body is $\sigma_{xx} = F/A$ and the strain is $\varepsilon_{xx} = \Delta L/L$. Plotting this, we find ---> That is, for one-dimensional rod stretching and *for small enough strain*, the stress and strain are linearly related:



$$\sigma_{xx} = E\varepsilon_{xx} \qquad (1)$$

where E is called Young's modulus. This is called Hooke's law which is valid for small enough strains for rods made of a certain class of materials.

Now, what if the body had a more complicated state of stress and strain? For instance, in addition to normal stress/strain, we also had shearing or something more complicated? We take a leap of faith and generalize Hooke's law to state that the six independent stresses are related to the six independent strains linearly:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} \qquad (2)$$

where I have used engineering shear strains.

The justification for this is that experiments seem to bear this out for a large class of materials of engineering interest. Materials for which the above is true are called *linear elastic*. This leaves us with 36 elastic constants that we have to determine for a given material. That is much more than we would like to handle, so we make some more simplifications about the material. We will consider only those linear elastic materials which are *isotropic*. By this we mean that the material has the same properties in all directions. Such isotropic behavior is the result of some underlying symmetry of the material microstructure (at the crystalline atomic level, or else in an average sense). For such isotropic linear elastic materials, it turns out that that there are only two elastic constants. The relation between stresses and strains at any point in such materials is given by:

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & \frac{-\nu}{E} & 0 & 0 & 0 \\ \frac{-\nu}{E} & \frac{1}{E} & \frac{-\nu}{E} & 0 & 0 & 0 \\ \frac{-\nu}{E} & \frac{-\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+\nu)}{E} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E} \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{bmatrix}$$

(3)

where E is the Young's modulus and ν is the Poisson's ratio of the material.

Remarks:

(i) Take these material response relations as experimental facts. We cannot really completely derive these based only on theoretical considerations. Also note that the ones I have laid out above are true only for small strains, and only for some materials. Materials respond in a variety of ways, the most important (from an engineering standpoint) of which is the above. We will consider only isotropic linear elastic materials in this course.

(ii) Note that for isotropic materials, normal stresses are related to only normal strains and shear stresses are related to only shear strains. This is not necessarily true for other kinds of materials.

(iii) Examples of isotropic materials are most polycrystalline metals. Examples of materials which are *not* isotropic are wood, rolled sheet metal, and most composite materials, which have different material response in different directions. They may have lesser degrees of symmetry than isotropy, and may require upto 21 (one can show that the elasticity matrix above must be symmetric from energy considerations allowing only for a maximum of 21 independent constants) elastic material properties to completely describe them.